1 Data storage in the results files

This section describes how the data are stored in the different results files.
For vectors $x \in \mathbb{R}^3$, x_1, x_2, x_3 denote the cartesian coordinates of x and x_r, x_θ, x_φ denote the spherical coordinates of x.

1.1 Radially symmetric conductivity

The files of real radially symmetric conductivities, given by the codes **`radsym`** and **`fast_radsym`** have a header containing the word "radial" and the second line gives the number n_{r} of points on the radius $[0, 1]$ where the conductivity is computed. The following lines give the value of the radius followed by the the value of the conductivity. In other words, the file is given by

$$\begin{array}{cc}
radial & \\
n_{\mathrm{r}} & \\
r^1 & \sigma(r^1) \\
\vdots & \vdots \\
r^{n_{\mathrm{r}}} & \sigma(r^{n_{\mathrm{r}}})
\end{array}$$

1.2 Conductivities on mesh

Except for the codes **`radsym`** and **`fast_radsym`**, the conductivities are computed at the points of a Gmsh mesh. For real conductivities, the header of the file contains the number n of mesh points, then each line of the file contain the three coordinates of a mesh point followed by the value of the conductivity at this point. More precisely, if we denote $x^1, \dots, x^n$ the mesh points and σ the conductivity, the file is given by

$$\begin{array}{cccc}
n & & & \\
x_1^1 & x_2^1 & x_3^1 & \sigma(x^1) \\
\vdots & \vdots & \vdots & \vdots \\
x_1^n & x._2 & x_3^n & \sigma(x^n)
\end{array}$$

For real conductivities, the header of the file contains the number n of mesh points followed by the word "complex", then each line of the file contain the three coordinates of a mesh point followed by the value of the conductivity at this point given by first the real part and then the imaginary part. More precisely, if we denote $x^1, \dots, x^n$ the mesh points and σ the conductivity, the file is given by

$$\begin{array}{ccccc}
n & \text{complex} & & & \\
x_1^1 & x_2^1 & x_3^1 & \Re\sigma(x^1) & \Im\sigma(x^1) \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
x_1^n & x_2^n & x_3^n & \Re\sigma(x^n) & \Im\sigma(x^n)
\end{array}$$

Remark 1.1. $q = \dfrac{\Delta\sigma^{\frac{1}{2}}}{\sigma^{\frac{1}{2}}}$ *is stored in the same way.*

Remark 1.2. *To read or write a conductivity or q in this format, you can use the* `Matlab` *functions* `read_conductivity` *and* `read_q`*.*
The functions `plot_conductivity` *and* `plot_q` *can be used to plot the data.*

1.3 Fourier Transform $\hat{q}$ of $q = \dfrac{\Delta\sigma^{\frac{1}{2}}}{\sigma^{\frac{1}{2}}}$

The Fourier Transform $\hat{q}$ is computed on a regular cubic grid $[-\xi_{\mathrm{max}}, \xi_{\mathrm{max}}]^3$ of n_{g}^3 points. The header of the Fourier Transform file contains n_{g} followed by ξ_{max}. The the lines of the file contains the coordinates of the grid points followed by the value of the Fourier Transform given by first the real part and then the imaginary part. The order the grid points $\xi^{\ell m,n} = (-\xi_{\mathrm{max}} + (\ell - 1)\dfrac{2\xi_{\mathrm{max}}}{n_{\mathrm{g}} - 1}, -\xi_{\mathrm{max}} + (m-1)\dfrac{2\xi_{\mathrm{max}}}{n_{\mathrm{g}} - 1}, -\xi_{\mathrm{max}} + (n-1)\dfrac{2\xi_{\mathrm{max}}}{n_{\mathrm{g}} - 1})\ ,\ \ell, m, n = 1 \dots n_{\mathrm{g}}$ in the file is given by the nested loops

```
for (unsigned $\ell$=1;$\ell$<=$n_{\mathrm{g}}$;$\ell$++)
  {
    for (unsigned $m$=1;$m$<=$n_{\mathrm{g}}$;$m$++)
      {
        for (unsigned $n$=1;$n$<=$n_{\mathrm{g}}$;$n$++)
          {
            $\xi^{\ell m n}$;
          }
      }
  }
```

Simply denoting by ξ^p the p-th ξ value, that is $p = n_{\mathrm{g}}^2(\ell - 1) + n_{\mathrm{g}}(m-1) + n$, the file is given by

$$\begin{array}{ccccc}
n_{\mathrm{g}} & \xi_{\mathrm{max}} & & & \\
\xi_1^1 & \xi_2^1 & \xi_3^1 & \Re\hat{q}(\xi^1) & \Im\hat{q}(\xi^1) \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\xi_1^{n_{\mathrm{g}}^3} & \xi_2^{n_{\mathrm{g}}^3} & \xi_3^{n_{\mathrm{g}}^3} & \Re\hat{q}(\xi^{n_{\mathrm{g}}^3}) & \Im\hat{q}(\xi^{n_{\mathrm{g}}^3})
\end{array}$$

Remark 1.3. *To read a $\hat{q}$ file in this format, you can use the* `Matlab` *function* `read_qhat`*.*
The functions allow to store the data either as a `ndgrid` *or a* `meshgrid`*.*

1.4 Real matrices

Except the header at the beginning of each file, the real matrices are stored in a straightforward way. Denote by n_{d} the maximal degree of spherical harmonics used for the computations, so that the number of quadrature points on the sphere is given by $N_{\mathrm{d}} = 2(n_{\mathrm{d}}+1)^2$. The header of the file for such matrices (Dirichlet-to-Neumann map for a conductivity 1, Dirichlet-to-Neumann map for a real conductivity, Single Layer Operator matrix for the usual Green's function) contains n_d and then the lines of the file give the matrix rows. Hence, after the header, we have N_d lines of N_d real numbers. More precisely, for a real matrix M corresponding to computations for n_d, the file is given by

$$\begin{array}{ccc} n_\mathrm{d} & & \\ M_{1\,1} & \dots & M_{1\,N_\mathrm{d}} \\ \vdots & \vdots & \vdots \\ M_{N_\mathrm{d}\,1} & \dots & M_{N_\mathrm{d}\,N_\mathrm{d}} \end{array}$$

1.5 Complex matrices

The header of the file for such matrices (Dirichlet-to-Neumann map for a complex conductivity) contains n_d followed by the word "complex" and then the lines of the file give the matrix rows. For each value of the matrix, the real part is given, followed by the imaginary part. Hence, after the header, we have N_d lines of $2N_\mathrm{d}$ real numbers. More precisely, for a complex matrix M corresponding to computations for n_d, the file is given by

$$\begin{array}{cccccc} n_\mathrm{d} & \text{complex} & & & & \\ \Re M_{1\,1} & \Im M_{1\,1} & \dots & \dots & \Re M_{1\,N_\mathrm{d}} & \Im M_{1\,N_\mathrm{d}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Re M_{N_\mathrm{d}\,1} & \Im M_{N_\mathrm{d}\,1} & \dots & \dots & \Re M_{N_\mathrm{d}\,N_\mathrm{d}} & \Im M_{N_\mathrm{d}\,N_\mathrm{d}} \end{array}$$

Remark 1.4. *To read or write a matrix file in this format, you can use the* **`Matlab`** *functions* **`read_dnmap`** *and* **`write_dnmap`**. *Despite of the name, they work the same to read or write a file for the Single Layer Operator matrix for the usual Green's function. If you want to add noise to a Dirichlet-to-Neumann map matrix, you can use the function* **`add_noise_dnmap`**. *You first read the non-noisy Dirichlet-to-Neumann map matrix file using* **`read_dnmap`**, *the you use the function* **`add_noise_dnmap`** *to add noise to the matrix, and finally you write the noisy Dirichlet-to-Neumann map matrix in a file using* **`write_dnmap`**.

1.6 Dirichlet-to-Neumann map eigenvalues

These data files concern the codes **`radsym`** and **`fast_radsym`**. The header gives the number n_e of non-zero computed eigenvalues followed by the discretization step h_r of the radius $[0,1]$ used to compute these eigenvalues. The next lines give the $n_\mathrm{e}+1$ first increasing eigenvalues (including 0) of the Dirichlet-to-Naumann map, followed by an estimate of the relative error between the computed eigenvalue and the true one, hence the file is given by

$$\begin{array}{cc} n_\mathrm{e} & h_\mathrm{r} \\ 0 & 0 \\ \lambda^1 & \varepsilon_{\lambda^1} \\ \vdots & \vdots \\ \lambda^{n_\mathrm{e}} & \varepsilon_{\lambda^{n_\mathrm{e}}} \end{array}$$

1.7 CGO solutions traces for the eit programs

The files storing the CGO solutions traces considered here, concern the codes **`eit`**, **`fast_eit`**, **`eit_cmplx`** and **`fast_eit_cmplx`**. The header of a CGO solutions traces file gives the maximal degree n_{d} of spherical harmonics used for the computations, so that the number of quadrature points on the sphere is given by $N_{\mathrm{d}} = 2(n_{\mathrm{d}}+1)^2$, followed by n_{g} and $\xi_{\max}$ giving the parameter regular cubic grid $[-\xi_{\max}, \xi_{\max}]^3$ of n_{g}^3 points. The second line gives the spherical coordinates of the N_{d} quadrature points on the unit sphere. We recall that the quadrature points on the sphere are given by $x^{m\,n} = (\sin\theta_m \cos\varphi_n, \sin\theta_m \sin\varphi_n, \cos\theta_m)$, $m = 0 \dots n_{\mathrm{d}}$, $n = 0 \dots 2n_{\mathrm{d}}+1$ where $\theta_m = \arccos t_m$, $m = 0 \dots n_{\mathrm{d}}$ and $\varphi_n = \pi n/(n_{\mathrm{d}}+1)$, $n = 0 \dots 2n_{\mathrm{d}}+1$, where t_m are the $n_{\mathrm{d}}+1$ increasing zeros of the Legendre polynomial $P_{n_{\mathrm{d}}+1}$. For all the computations, as well as in the CGO solutions traces file, these points are ordered according to the nested loops

```
for (unsigned $n$=0;$n$<=$2 * n_{\mathrm{d}} + 1$;$n$++)
  {
    for (unsigned $m$=0;$m$<=$n_{\mathrm{d}} + 1$;$m$++)
      {
        $x^{m\,n}$;
      }
  }
```

The next lines give the coordinates of ξ followed by the coordinates of the corresponding chosen ζ, each coordinate of the latter given by the real part followed by the imaginary part. Each line ends with the values of the CGO solution trace ψ_ζ at the quadrature points, each value of the CGO solution trace given by the real part followed by the imaginary part. The order of ξ values is as in subsection 1.3. In other words, the file is given by

$$\begin{array}{ccccccccccccccc}
n_{\mathrm{d}} & n_{\mathrm{g}} & \xi_{\max} & & & & & & & & & & & & \\
x_\theta^1 & x_\varphi^1 & \dots & \dots & x_\theta^{N_{\mathrm{d}}} & x_\varphi^{N_{\mathrm{d}}} & & & & & & & & & \\
\xi_1^1 & \xi_2^1 & \xi_3^1 & \Re\zeta_1^1 & \Im\zeta_1^1 & \Re\zeta_2^1 & \Im\zeta_2^1 & \Re\zeta_3^1 & \Im\zeta_3^1 & \Re\psi_{\zeta^1}(x^1) & \Im\psi_{\zeta^1}(x^1) & \dots & \dots & \Re\psi_{\zeta^1}(x^{N_{\mathrm{d}}}) & \Im\psi_{\zeta^1}(x^{N_{\mathrm{d}}}) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\xi_1^{n_{\mathrm{g}}^3} & \xi_2^{n_{\mathrm{g}}^3} & \xi_3^{n_{\mathrm{g}}^3} & \Re\zeta_1^{n_{\mathrm{g}}^3} & \Im\zeta_1^{n_{\mathrm{g}}^3} & \Re\zeta_2^{n_{\mathrm{g}}^3} & \Im\zeta_2^{n_{\mathrm{g}}^3} & \Re\zeta_3^{n_{\mathrm{g}}^3} & \Im\zeta_3^{n_{\mathrm{g}}^3} & \Re\psi_{\zeta^{n_{\mathrm{g}}^3}}(x^1) & \Im\psi_{\zeta^{n_{\mathrm{g}}^3}}(x^1) & \dots & \dots & \Re\psi_{\zeta^{n_{\mathrm{g}}^3}}(x^{N_{\mathrm{d}}}) & \Im\psi_{\zeta^{n_{\mathrm{g}}^3}}(x^{N_{\mathrm{d}}})
\end{array}$$

Remark 1.5. *To read a file in this format, you can use the* `Matlab` *function* **`read_cgostraces`**. *To plot the CGO solutions traces, you can use the function* **`plot_cgostraces`**.

1.8 CGO solutions traces for the cgos program

The files storing the CGO solutions traces considered here, concern the code **`cgos`**. The header of a CGO solutions traces file gives the maximal degree n_{d} of spherical harmonics used for the computations, so that the number of quadrature points on the sphere is given by $N_{\mathrm{d}} = 2(n_{\mathrm{d}}+1)^2$, followed by $\kappa = |\zeta|/\sqrt{2}$ and by n_k, n_θ, n_φ. Writing $\zeta = \kappa(k^\perp + ik^{\perp\perp})$, with $|k^\perp| = |k^{\perp\perp}| = 1$ and $k^\perp \cdot k^{\perp\perp} = 0$, n_k gives the number of discretization points of $[0, 2\pi)$ for the rotation of $k^\perp$ around k, n_θ gives the number of discretization points of

$[0,\pi)$ for the elevation of $k^{\perp\perp}$, n_θ gives the number of discretization points of $[0,\pi)$ for the azimuth of $k^{\perp\perp}$. ζ/κ is given in the storing file by the rotation angle η of $k^{\perp}$ around $k^{\perp\perp}$, followed by the spherical coordinates (θ,φ) of $k^{\perp\perp}$. The angle η is measured with respect to the vector of spherical coordinates $(\theta+\pi/2,\varphi)$. In other words, we have

$$k^{\perp\perp} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}$$

$$k^{\perp} = \begin{pmatrix} \cos\eta\cos\theta\cos\varphi - \sin\eta\sin\varphi \\ \cos\eta\cos\theta\sin\varphi + \sin\eta\cos\varphi \\ -\cos\eta\sin\theta \end{pmatrix} .$$

The second line gives the spherical coordinates of the N_d quadrature points on the unit sphere. We recall that the quadrature points on the sphere are given by
$x^{m\,n} = (\sin\theta_m\cos\varphi_n, \sin\theta_m\sin\varphi_n, \cos\theta_m)$, $m = 0\ldots n_\mathrm{d}$, $n = 0\ldots 2n_\mathrm{d}+1$ where $\theta_m = \arccos t_m$, $m = 0\ldots n_\mathrm{d}$ and $\varphi_n = \pi n/(n_\mathrm{d}+1)$, $n = 0\ldots 2n_\mathrm{d}+1$, where t_m are the $n_\mathrm{d}+1$ increasing zeros of the Legendre polynomial $P_{n_\mathrm{d}+1}$. For all the computations, as well as in the CGO solutions traces file, these points are ordered according to the nested loops

```
for (unsigned $n$=0;$n$<=$2*n_\mathrm{d}+1$;$n$++)
  {
    for (unsigned $m$=0;$m$<=$n_\mathrm{d}+1$;$m$++)
      {
        $x^{m\,n}$;
      }
  }
```

The next lines give the angles η_ℓ , θ_m , φ_n , $\ell = 0\ldots n_k-1$, $m = 0\ldots n_\theta-1$, $n = 0\ldots n_\varphi-1$ for $k^{\perp}$ and $k^{\perp\perp}$ followed by the values of the CGO solution trace ψ_ζ at the quadrature points, each value of the CGO solution trace given by the real part followed by the imaginary part. The values of η_ℓ , θ_m , φ_n , $\ell = 0\ldots n_k-1$, $m = 0\ldots n_\theta-1$, $n = 0\ldots n_\varphi-1$ are ordered according to the loop and nested loops

```
for (unsigned $\ell$=0;$\ell$<$n_k$;$\ell$++)
  {
    $\eta_\ell, 0, 0$;
  }
for (unsigned $n$=0;$n$<$n_\varphi$;$n$++)
  {
    for (unsigned $m$=1;$m$<$n_\theta$;$m$++)
      {
        for (unsigned $\ell$=0;$\ell$<$n_k$;$\ell$++)
          {
            $\eta_\ell, \theta_m, \varphi_n$;
          }
      }
  }
```

In other words, denoting by $n_{\mathrm{g}} = n_k\left((n_\theta - 1)n_\varphi + 1\right)$ the number of values for ζ, the file is given by

$$\begin{array}{ccccccccc}
n_{\mathrm{d}} & \kappa & n_k & n_\theta & n_\varphi & & & & \\
x_\theta^1 & x_\varphi^1 & \dots & \dots & x_\theta^{N_{\mathrm{d}}} & x_\varphi^{N_{\mathrm{d}}} & & & \\
\eta^1_{k^\perp} & \theta^1_{k^{\perp\perp}} & \varphi^1_{k^{\perp\perp}} & \Re\psi_{\zeta^1}(x^1) & \Im\psi_{\zeta^1}(x^1) & \dots & \dots & \Re\psi_{\zeta^1}(x^{N_{\mathrm{d}}}) & \Im\psi_{\zeta^1}(x^{N_{\mathrm{d}}}) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\eta^{n_{\mathrm{g}}}_{k^\perp} & \theta^{n_{\mathrm{g}}}_{k^{\perp\perp}} & \varphi^{n_{\mathrm{g}}}_{k^{\perp\perp}} & \Re\psi_{\zeta^{n_{\mathrm{g}}}}(x^1) & \Im\psi_{\zeta^{n_{\mathrm{g}}}}(x^1) & \dots & \dots & \Re\psi_{\zeta^{n_{\mathrm{g}}}}(x^{N_{\mathrm{d}}}) & \Im\psi_{\zeta^{n_{\mathrm{g}}}}(x^{N_{\mathrm{d}}})
\end{array}$$

Remark 1.6. *To read a file in this format, you can use the* `Matlab` *function* `read_cgos_cgostraces`*.*